

Claims:**1. An arrangement comprising:**

A collection of filters responsive to a common signal, where filter i has the

5 transfer function $V(e^{j(\omega - \frac{2\pi}{N}i)})$, V is a given low pass filter function, $i=0,1,2,\dots,N-1$,

and $\sum_{i=0}^{N-1} \left| V(e^{j(\omega - \frac{2\pi}{N}i)}) \right|^2 = M$ for all $\omega \in (-\pi, \pi)$.

2. An arrangement comprising:

a collection of filters, each responsive to a different signal, where filter i has

10 the transfer function $V^*(e^{j(\omega - \frac{2\pi}{N}i)})$, and V is a time-reversed version of a low pass filter V ; and

a combiner for adding output signals of said filters to form an output signal of said arrangements.

15 **3. The arrangement of claim 2 where** $\sum_{i=0}^{N-1} \left| V^*(e^{j(\omega - \frac{2\pi}{N}i)}) \right|^2 = N$ for all $\omega \in (-\pi, \pi)$.

4. An arrangement comprising:

an n_1 plurality of analysis filters i , where $i=0,1,\dots,n_1-1$, each developing an output signal and each coupled to a circuit that subsamples the associated output

20 signal by a factor of K_1 , where each filter i has a transfer function $V_1(e^{j(\omega - \frac{2\pi}{N_1}i)})$, symmetric about $\omega = 0$, where n_1 is a first constant, N_1 is a second constant, and V_1 is a first preselected low pass filter function;

an $(N_2/2) - n_2$ plurality of analysis filters k where $k = n_2+1, n_2+2, \dots, N_2/2$, each developing an output signal and each coupled to a circuit that subsamples the

25 associated output signal by a factor of K_2 , where each filter k has a transfer

function $V_2(e^{j(\omega - \frac{2\pi k}{N_2})})$, symmetric about $\omega = 0$, where n_2 is a third constant, N_2 is a fourth constant, and V_2 is a second preselected low pass filter function; and

a transition analysis filter that develops an output signal, coupled to a circuit that subsamples said output signal of said transition filter by a factor of K_1 or K_2 ,
 5 having a bandpass transfer function that fills-in a gap frequencies that are passed by said n_1 plurality of filters and frequencies that are passed by said $(N_2/2) - n_2$ plurality of filters.

5. The arrangement of claim 4 where V_1 has an attenuation at frequencies
 10 greater than $2\pi/N_1$ that is large enough to produce an insignificant output at those frequencies, and V_2 has an attenuation at frequencies greater than $2\pi/N_2$ that is large enough to produce an insignificant output at those frequencies.

6. The arrangement of claim 4 where V_1 is such that an energy integral
 15 $\int_{\pi/N_1}^{\pi} |V_1(e^{j\omega})|^2 \omega^3 d\omega$ that is associated with V_1 is lower than a preselected level.

7. The arrangement of claim 4 where $N_2 < N_1$.

8. The arrangement of claim 4 where said transition analysis filter has a
 20 passband between $\frac{\pi(2n_1-1)}{N_1}$ and $\frac{\pi(2n_2+1)}{N_2}$.

9. The arrangement of claim 4 where said transition analysis filter has a
 transfer function $V_{1,2}(e^{j(\omega+\omega_0)})$, where $\omega_0 = 2\pi n_1/N_1 = 2\pi n_2/N_2$ and $V_{1,2}$ is a third low
 pass function.

25 10. The arrangement of claim 9 where $V_{1,2}$ is a low pass filter with a transfer function that, at positive frequencies, corresponds to said transfer function

V_2 at positive frequencies, and at negative frequencies corresponds to said transfer function V_1 at negative frequencies.

11. The arrangement of claim 9 where window shape of $V_{1,2}$ satisfies

5 condition

$$\frac{1}{K_1} \sum_{i=0}^{n_1-1} \left| V_1(e^{j(\omega + \frac{2\pi}{N_1}i)}) \right|^2 + \frac{1}{K_2} \left| V_{1,2}(e^{j(\omega + \omega_0)}) \right|^2 + \frac{1}{K_2} \sum_{i=n_2}^{N_2-1} \left| V_2(e^{j(\omega + \frac{2\pi}{N_2}i)}) \right|^2 \approx R$$

when said subsampling at the circuit coupled to said transition analysis filter subsamples by a factor of K_2 , and the condition

$$\frac{1}{K_1} \sum_{i=0}^{n_1-1} \left| V_1(e^{j(\omega + \frac{2\pi}{N_1}i)}) \right|^2 + \frac{1}{K_1} \left| V_{1,2}(e^{j(\omega + \omega_0)}) \right|^2 + \frac{1}{K_2} \sum_{i=n_2}^{N_2-1} \left| V_2(e^{j(\omega + \frac{2\pi}{N_2}i)}) \right|^2 \approx R$$

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when said subsampling at the circuit coupled to said transition analysis filter subsamples by a factor of K_1 , where $R = N_1 / K_1 = N_2 / K_2$.

12. The arrangement of claim 9 where $V_{1,2}$ is a function such that an

energy integral $E_{tr} = \int_{-\pi}^0 \left| V_1(e^{j\omega}) - V_{1,2}(e^{j\omega}) \right|^2 d\omega + \int_0^{\pi} \left| \beta V_2(e^{j\omega}) - V_{1,2}(e^{j\omega}) \right|^2 d\omega$ that is

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associated with $V_{1,2}$ is minimized, where $\beta = \sqrt{N_1 / N_2}$.

13. The arrangement of claim 4 where said transition analysis filter has a

transfer function $V_{1,2}(e^{j(\omega + \omega_0)})$, where $2\pi(n_1 - 1) / N_1 < \omega_0 < 2\pi(n_2 + 1) / N_2$ and $V_{1,2}$ is a

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third low pass function.

14. The arrangement of claim 13 where $V_{1,2}$ is a low pass filter with a

transfer function that, at positive frequencies, corresponds to said transfer function

V_2 at positive frequencies, and at negative frequencies corresponds to said

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transfer function V_1 at negative frequencies.

15. The arrangement of claim **13** where window shape of $V_{1,2}$ satisfies condition

$$\frac{1}{K_1} \sum_{i=0}^{n_1-1} \left| V_1(e^{j(\omega + \frac{2\pi}{N_1}i)}) \right|^2 + \frac{1}{K_2} \left| V_{1,2}(e^{j(\omega + \omega_0)}) \right|^2 + \frac{1}{K_2} \sum_{i=n_2}^{N_2-1} \left| V_2(e^{j(\omega + \frac{2\pi}{N_2}i)}) \right|^2 \approx R$$

when said subsampling at the circuit coupled to said transition analysis filter

5 subsamples by a factor of K_2 , and the condition

$$\frac{1}{K_1} \sum_{i=0}^{n_1-1} \left| V_1(e^{j(\omega + \frac{2\pi}{N_1}i)}) \right|^2 + \frac{1}{K_1} \left| V_{1,2}(e^{j(\omega + \omega_0)}) \right|^2 + \frac{1}{K_2} \sum_{i=n_2}^{N_2-1} \left| V_2(e^{j(\omega + \frac{2\pi}{N_2}i)}) \right|^2 \approx R$$

when said subsampling at the circuit coupled to said transition analysis filter subsamples by a factor of K_1 , where $R = N_1 / K_1 = N_2 / K_2$.

10 **16.** An arrangement comprising:

an n_1 plurality of upsampling circuits, each responsive to an associated input signal and upsampling its input signal by a factor of K_1 ;

an n_1 plurality of synthesis filters i where $i=0,1,\dots,n_1-1$, each responsive to an associated one of said n_1 plurality of upsampling circuits, where each filter i has

15 a transfer function $V_1(e^{j(\omega + \frac{2\pi}{N_1}i)})$, symmetric about $\omega = 0$, n_1 is a first constant, N_1 is a second constant, and V_1 is a first preselected low pass filter function ;

an $(N_2/2) - n_2$ plurality of upsampling circuits, each responsive to an associated input signal port and upsampling its input signal by a factor of K_2 ;

an $(N_2/2) - n_2$ plurality of synthesis filters k where $k = n_2+1, n_2+2, \dots, N_2/2$,

20 each responsive to an associated one of said $(N_2/2) - n_2$ plurality of upsampling

circuits, where each filter k has a transfer function $V_2(e^{j(\omega + \frac{2\pi}{N_2}k)})$, symmetric about $\omega = 0$, n_2 is a third constant, N_2 is a fourth constant, and V_2 is a second preselected low pass filter function;

a transition upsampling circuit, responsive to an associated input signal port
25 that oversamples its input signal by a factor of K_1 or K_2 ;

a transition synthesis filter responsive to said transition upsampling circuit, having a bandpass transfer function that fills-in a gap frequencies passed by said n_1 plurality of filters and frequencies passed by said $(N_2/2) - n_2$ plurality of filters; and

5 an addition circuit that combines output signals of said n_1 plurality of filters, said transition filter, and said $(N_2/2) - n_2$ plurality of filters, to develop an output signal.

10 **17.** The arrangement of claim **16** where V_1 has an attenuation at frequencies greater than $2\pi / N_1$ that is large enough to produce an insignificant output at those frequencies, and V_2 has an attenuation at frequencies greater than $2\pi / N_2$ that is large enough to produce an insignificant output at those frequencies.

15 **18.** The arrangement of claim **16** where V_1 is such that an energy integral $\int_{\pi/N_1}^{\pi} |V_1^{\%}(e^{j\omega})|^2 \omega^3 d\omega$ that is associated with V_1 is lower than a preselected level.

19. The arrangement of claim **16** where $N_2 < N_1$.

20 **20.** The arrangement of claim **16** where said transition synthesis filter has a pass band between $\frac{\pi(2n_1-1)}{N_1}$ and $\frac{\pi(2n_2+1)}{N_2}$.

21. The arrangement of claim **16** where said transition synthesis filter has a transfer function $V_{1,2}^{\%}(e^{j(\omega+\omega_0)})$, where $\omega_0 = 2\pi n_1 / N_1 = 2\pi n_2 / N_2$ and $V_{1,2}^{\%}$ is a third low
25 pass function.

22. The arrangement of claim **16** where said transition synthesis filter has a transfer function $V_{1,2}^{\%}(e^{J(\omega+\omega_0)})$, where $2\pi(n_1-1)/N_1 < \omega_0 < 2\pi(n_2+1)/N_2$ and $V_{1,2}^{\%}$ is a third low pass function.

23. The arrangement of claim **21** where $V_{1,2}^{\%}$ is a function such that an energy integral $E_{tr} = \int_{-\pi}^0 |V_1^{\%}(e^{j\omega}) - V_{1,2}^{\%}(e^{j\omega})|^2 d\omega + \int_0^{\pi} |\beta V_2^{\%}(e^{j\omega}) - V_{1,2}^{\%}(e^{j\omega})|^2 d\omega$ that is associated with $V_{1,2}^{\%}$ is minimized, where $\beta = \sqrt{N_1/N_2}$.

24. The arrangement of claim **21** where $V_{1,2}^{\%}$ is a low pass filter with a transfer function that, at positive frequencies, corresponds to said transfer function V_2 at positive frequencies, and at negative frequencies corresponds to said transfer function V_1 at negative frequencies.

25. The arrangement of claim **21** where window shape of $V_{1,2}^{\%}$ satisfies condition

$$\frac{1}{K_1} \sum_{l=0}^{n_1-1} \left| V_1^{\%}(e^{J(\omega+\frac{2\pi}{N_1}l)}) \right|^2 + \frac{1}{K_2} |V_{1,2}^{\%}(e^{J(\omega+\omega_0)})|^2 + \frac{1}{K_2} \sum_{l=n_2}^{N_2-1} \left| V_2^{\%}(e^{J(\omega+\frac{2\pi}{N_2}l)}) \right|^2 \approx R \text{ when said}$$

subsampling at the circuit coupled to said transition synthesis filter subsamples by a factor of K_2 , and the condition

$$\frac{1}{K_1} \sum_{l=0}^{n_1-1} \left| V_1(e^{J(\omega+\frac{2\pi}{N_1}l)}) \right|^2 + \frac{1}{K_1} |V_{1,2}(e^{J(\omega+\omega_0)})|^2 + \frac{1}{K_2} \sum_{l=n_2}^{N_2-1} \left| V_2(e^{J(\omega+\frac{2\pi}{N_2}l)}) \right|^2 \approx R$$

when said subsampling at the circuit coupled to said transition analysis filter subsamples by a factor of K_1 , where $R = N_1/K_1 = N_2/K_2$.

26. The arrangement of claim **4** comprising:

a signal processing stage responsive to output signals of said n_1 plurality of analysis filters, said $(N_2/2) - n_2$ plurality of analysis filters, and said transition analysis filter, and developing therefrom a plurality of processed signals; and
 an arrangement according to claim 16 responsive to said plurality of
 5 processed signals.

27. The arrangement of claim 26 where said plurality of processed signals comprises n_1 processed signals that are respectively derived from output signals of said n_1 plurality of analysis filters, a processed signal that is derived from said
 10 transition analysis filter, and $(N_2/2) - n_2$ processed signals that are respectively derived from said $(N_2/2) - n_2$ plurality of analysis filters.

28. The arrangement of claim 27 where
 a processed signal derived from filter i of said n_1 plurality of analysis filters
 15 is coupled to the upsampling circuit that is coupled to filter i of said n_1 plurality of synthesis filters;

a processed signal derived from filter k of said $(N_2/2) - n_2$ plurality of analysis filters is coupled to the upsampling circuit that is coupled to filter k of said
 20 $(N_2/2) - n_2$ plurality of synthesis filters; and

said processed signal derived from said transition analysis filter is coupled
 to the upsampling circuit that is coupled to said transition synthesis filter.

29. The arrangement of claim 28 where V_q is a low pass filter, q takes on the value 1, 2, or 1,2, and V_q a time-reversed version of said filter V_q .

30. A filter arrangement comprising:

P filter banks, each of which covers a distinct, non-overlapping, frequency band, leaving a frequency gap between a frequency where a pass band of one of said P filter banks ends and a pass band of any another one of said P filter banks
 30 begins, thus forming $P-1$ frequency gaps, and where each filter bank includes a

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